Solutions

Final Exam Review
Linear and Exponential Word Problems

Writing Linear and Exponential Models: For the following four problems, take the information given and write first a linear model and, second, write an exponential model with the given parameters.

1. Helen is draining water from her pool. When she starts, the pool has 100 gallons of water in it and 1 minute later there is 95 gallons. Write your models with respect to time measured in minutes.

$95-100$

Linear: $f(t) = -5t + 100$

$\frac{95}{100}$

Exponential: $g(t) = 100 \cdot (0.95)^t$

2. George is running a guinea pig farm and he has 120 guinea pigs. After one year, he has 150 guinea pigs. Write your models with respect to time measured in years.

$150-120$

Linear: $f(t) = 120 + 30t$

$\frac{150}{120}$

Exponential: $g(t) = 120 \cdot (1.25)^t$

3. Ash Ketchum starts his Pokemon journey with a single pokemon, his beloved Pikachu. After three month of traveling the wilds, he has managed to acquire 9 pokemon. Write your models with respect to time measure in months.

$\frac{9-1}{3-0}$

Linear: $f(t) = \frac{8}{3}t + 1$

$\left(\frac{9}{1}\right)^{1/3}$

Exponential: $g(t) = 1 \cdot 9^{t/3}$

4. A spaceship is traveling the stars in a straight line away from the earth. Two months after the journey begins, the spaceship is 10 light years away from Earth. One month later, the spaceship is 15 lights years away from Earth. Write your models with respect to time measured in months.

$\frac{15-10}{3-2}$

Linear: $f(t) = 5t$

$\frac{15}{10}$

Exponetial: $g(t) = 10 \cdot (1.5)^{t-2}$

$= \frac{10}{(1.5)^2} \cdot (1.5)^t$

Linear or Exponential? For the remaining problems, you must decide whether an exponential or a linear model is appropriate.

5. Mary decides to invest \$3000 into a bank account which gains 4% interest compounded annually.

(a) Write an appropriate model for the amount of money in the account after t years.

(b) How much money is in the account after 10 years?

(c) How long will it take Mary to have \$7000 in her account?

(a) $A(t) = 3000(1.04)^t$

(b) $A(10) = 3000(1.04)^{10}$ dollars

(c) $7000 = 3000(1.04)^t \Rightarrow \frac{7}{3} = (1.04)^t$

$t = \frac{\ln(7/3)}{\ln(1.04)}$ Years

6. Miner Bob is pickaxeing his way to what he believes to be, a literal and metaphorical, goldmine. Assume that he can clear 7 meters with 10 minutes of pickaxeing.

(a) Write an appropriate model for the amount of mine Bob clears measured in t minutes.

(b) Bob's goldmine lies 100 meters away from his current position. How long will it take him to clear his way to it?

(c) What percentage of the way will Bob be to his destination in one hour?

(a) $f(t) = \frac{7}{10} \cdot t$

(b) $100 = \frac{7}{10} \cdot t \Rightarrow t = \frac{1000}{7}$ minutes

(c) $f(60) = \frac{7}{10} \cdot 60 = 42$ meters $\quad \frac{42}{100} = 42\%$.

7. Jason and Phil are both traveling to school from there shared home. Jason leaves home and travels at a rate of 35 mph. Phil leaves home 15 minutes after Jason, but he knows that he is late and so he speeds at a rate of 50 mph.

(a) Write appropriate models for Jason's and Phil's distance from home t minutes after Jason leaves.

(b) How long will it take for Phil to pass Jason?

(c) Assuming that school is 100 miles away, who will reach school first?

(a) Jason: $f(t) = \frac{35}{60} t = \frac{7}{12} t$

Phil: $\frac{50}{60} = \frac{5}{6}$ miles per minute.

$$0 = g(15) = \frac{5}{6}(15) + b \Rightarrow b = -12.5$$

$$g(t) = \frac{5}{6} t - 12.5$$

(b) $\frac{7}{12} t = \frac{5}{6} t - 12.5$

$$12.5 = \frac{3}{12} t \Rightarrow t = 50 \text{ minutes.}$$

(c) $f(50) = \frac{7}{12} \cdot 50 < 100$ so

Phil reaches school first.

8. Sour Patch Kids is trying to model the supply and demand curves for their product. They have discovered that the supply curve grows at a rate of 7% per 10 cents they increase the price. Further, they have discovered the the demand curve decreases at a rate of 9% per 10 cents they increase the price.

(a) Assume that at a price of 0 cents Sour Patch Kids supplies 3 boxes of candy. Write an appropriate model for the supply curve.

(b) Assume that at a price of 0 cents the consumers demand 300 boxes of candy. Write an appropriate model for the demand curve.

(c) Using the two models from parts (a) and (b), find the price at which supply and demand are equal. (This is called the equilibrium price.

(a) $S(t)=3(1.07)^t$ (measured in 10's of cents)

(b) $D(t)=300(0.91)^t$ (measured in 10's of cents)

(c) $S(t)=D(t)$

$$\frac{3(1.07)^t}{3(0.91)^t}=\frac{300(0.91)^t}{3(0.91)^t}$$

$$\left(\frac{1.07}{0.91}\right)^t=100$$

$$\log\left(\left(\frac{1.07}{0.91}\right)^t\right)=\log(100)$$

$$t\log\left(\frac{1.07}{0.91}\right)=2$$

$$t=\frac{2}{\log\left(\frac{1.07}{0.91}\right)}$$

10's of cents.